Comparative Study of the Insulator-Hall Liquid-Insulator Transitions: Composite Boson Picture vs Composite Fermion Picture

Wenjun Zheng, Yue Yu and Zhaobin Su
Institute of Theoretical Physics, Chinese Academy of Sciences, Beijing 100080, P.R.China
(February 1, 2008)

Based on a newly advanced phenomenological understanding of the high-field insulator- Hall liquid transition in a composite fermion picture, we extend its composite boson counterpart to the analysis of the low-field insulator-Hall liquid transition. We thus achieve a comparative study of these two transitions. In this way, the similar reflection symmetries in filling factors in both transitions are understood consistently as due to the symmetry of the gapful excitations which dominate σ_{xx} across the transitions, and the abrupt change in σ_{xy} at the transitions. The substantially different characteristic energy scales involved in these two transitions can be attributed to the differences in critical filling factors ν_c and the effective masses. The opposite temperature-dependences of the critical longitudinal resistivities are also well-understood, which can be traced to the opposite statistical natures of the composite fermion and the composite boson. We also give a tentative discussion of the zero-temperature dissipative conductivity. The above results are supported by a recent experiment (cond-mat/9708239).

PACS numbers: 73.40H, 71.30

The study of the magnetic field induced insulator-Hall liquid transitions has evoked considerable interests in the past decade. In the theoretical aspect, Kivelson, Lee and Zhang(KLZ) [1] started from the bosonic Chern-Simons field theory [2] and gave an elegant derivation which lead to the overall phase diagram of the general quantum Hall system with respect to disorder and magnetic field. A qualitatively identical phase diagram was obtained by Halperin, Lee and Read [3], from the celebrated composite fermion(CF) theory [4]. The well-known correspondence rule advanced by KLZ established a series of connections between the plateau-plateau transitions and the insulator-Hall liquid transitions, which suggests the possibility of super-universality [5] in the diverse quantum phase transitions observed in the quantum Hall system [6] [7]. Many experiments have been conducted to check these ideas [8].

In a recent experiment [9], Hilke et al. examined the magnetic field driven insulator-quantum Hall liquid-insulator transitions of the two dimensional hole system(2DHS) in a Ge/SiGe quantum well. With the increase of magnetic field, they found interesting similarities between the low-field(LF) insulator-Hall liquid transition and the high-field(HF) Hall liquid-insulator transition, with respect to the transport properties. First, the critical longitudinal resistivity at the LF transition ρ_c^L and the one at the HF transition ρ_c^H are approximately equal(Fig.1),

$$\rho_c^L = \rho_c^H \pm 3\%.$$

Secondly, there is a reflection symmetry in ρ_{xx} at the LF transition similar to the one at the HF transition previously reported by Shahar et al. [10]. These relations can be fitted by the following equations(Fig.2):

$$\rho_{xx}^L(\nu, T) = \rho_c^L \exp[\frac{\Delta\nu}{\alpha_0^L(T)}], \qquad (1)$$

where $\Delta\nu = \nu - \nu_c^L$, $\alpha_0^L(T) = \alpha^L T + \beta^L$, and α^L, β^L are sample-dependent parameters;

$$\rho_{xx}^H(\nu, T) = \rho_c^H \exp[\frac{-\Delta\nu}{\alpha_0^H(T)}], \qquad (2)$$

where $\Delta\nu = \nu - \nu_c^H$, $\alpha_0^H(T) = \alpha^H T + \beta^H$ and α^H, β^H are sample-dependent parameters;

In spite of the above similarities which suggest similar mechanisms for the two transitions, Hilke et al. also pointed out several differences between the HF transition and the LF transition: First, there are quite different characteristic energy scales involved in these two transitions, i.e.

$$\alpha^L/\alpha^H \approx 0.19/0.03 \approx 6,$$

as the measurement in ref [9] showed(see FIG.2). Secondly, the temperature-dependences of ρ_c^L and ρ_c^H are opposite, with the former decreasing with higher T while the later increasing with higher T. Both behaviors show up when T is larger than certain threshold values(FIG.1).

The similar properties of the LF transition and the HF transition seem to favor the floating up recipe [11], where both transitions are attributed to the crossing of the Fermi level with the lowest extended level. However, the substantially different energy scales and the qualitatively opposite T-dependences in the critical resistivities are beyond its predictions. It will be the aim of this paper to present a consistent phenomenological picture accounting for the above similarities and differences.

In a recent paper [12], we present a phenomenological picture based on the composite fermion theory, in order to understand the reflection symmetry near the transition from a $\nu = 1$ quantum Hall liquid to a Hall insulator (the above-mentioned HF transition). In that picture, the seemingly unexpected reflection symmetry in the longitudinal resistivity ρ_{xx} can be understood clearly as due

to the symmetry of the gapful excitations which dominate σ_{xx} across the transition, and the abrupt change in σ_{xy} at the transition. The parameter α in the linear fit of $\nu_0(T)$ in ref [10] is also given a simple physical meaning. Based on that theory the effective mass can be calculated from α, which gives a reasonable value of several electron band mass. When taking into account the previous network model calculations, the nearly invariant Hall resistivity ρ_{xy} across the transition is also well-understood.

One can see that the above picture does not directly depend on the statistical nature of the CFs. That is to say, the CB counterpart of it will produce the same reflection symmetry in ρ_{xx}. Based on this consideration, we attempt to use this CB picture to describe the LF transition, and then give an explanation of the properties that are different from the HF transition.

Noticing that the critical magnetic field at the HF transition $B_c^H = 4.04T$ is almost twice the value at the LF transition $B_c^L = 1.975T$, we argue that the one-flux-quanta bound composite boson will also be a kind of good quasiparticle near the LF transition, under the condition that the two-flux-quanta bound composite fermion is useful in accounting for the transport properties near the HF transition. Because the landau level (LL) mixing is more severe near the LF transition with relatively larger disorder and smaller LL spacing, the measured critical filling factor $\nu_c^L = 1.77$ deviates much from the ideal lowest LL constrained factor 1. We ignore here the possible contribution of the particles from the second LL. Therefore, we can suppose that at the LF transition, there are well-defined CBs in a zero effective magnetic field, while the CBs will feel the energy gap induced by the effective magnetic field B^* when ν deviates from ν_c^L a little. So we have

$$\sigma_{xx}(\nu,T) \propto \sigma_{xx}^{CB}(\nu,T) \propto \exp(-\omega_c^*/k_BT), \tag{3}$$

where $\omega_c^* = \frac{\hbar e B^*}{m_{CB}^*} \propto |\nu - \nu_c^L|$, $\hbar$ is set to unit, and the first relation is derived from the transformation between electrons and CBs given by ref [1].

In spite of the above continuity and symmetry around ν_c^L, there should be however a sharp change in the Hall conductivity σ_{xy} across the LF transition point. For $\nu < \nu_c^L$ or the quantum Hall liquid phase, we have $\sigma_{xy} = e^2/h$ $(T \to 0)$; while for $\nu > \nu_c^L$ or the insulator phase, we get $\sigma_{xy} \to 0$ $(T \to 0)$. This is also consistent with the well-known "floating up" recipe [11], where the QHL-Insulator transition occurs at the crossing of the Fermi level with the lowest extended state and σ_{xy} is determined by the number of extended states below the Fermi level.

With the above results of conductivity, We can obtain the resistivity tensor by conducting an inversion of the conductivity tensor, that is

$$\rho_{xx} = \frac{\sigma_{xx}}{\sigma_{xx}^2 + \sigma_{xy}^2}. \tag{4}$$

When $\Delta\nu < 0$,

$$\begin{aligned} \rho_{xx} &\approx \frac{\sigma_{xx}}{\sigma_{xy}^2} \\ &\propto \sigma_{xx} \\ &\propto \exp(-\omega_c^*/k_BT). \end{aligned} \tag{5}$$

When $\Delta\nu > 0$,

$$\begin{aligned} \rho_{xx} &\approx \sigma_{xx}^{-1} \\ &\propto \exp(\omega_c^*/k_BT). \end{aligned} \tag{6}$$

Combining the above results and the relation $\omega_c^* \propto |\Delta\nu|$, we can easily identify the reflection symmetry,

$$\rho_{xx}^L(\nu,T) = \rho_c^L \exp[\frac{\Delta\nu}{\alpha^L T}],$$

where

$$\alpha^L = \frac{2\pi k_B m_{CB}^* \nu_c^{L2}}{h^2 n}, \tag{7}$$

n is the density of the 2DHS. After taking into account the corresponding result for the HF transition (the case of CFs), we have

$$\frac{\alpha^L}{\alpha^H} = \frac{\nu_c^{L2}}{\nu_c^{H2}} \frac{m_{CB}^*}{m_{CF}^*}.$$

Since $\nu_c^L \approx 2\nu_c^H$ contributes a factor 4 in the ratio $\alpha^L/\alpha^H \approx 6$, we can attribute the substantially different energy scales between the LF transition and the HF transition as mainly due to the variant ν_c. Besides, we also expect $\frac{m_{CB}^*}{m_{CF}^*}$ to be larger than 1, because of the different extents of disorder for CFs and CBs(see below). In this respect the above theory is consistent with the experimental result of the ratio $\alpha^L/\alpha^H \approx 6$.

In another way, one can use the data from ref [9] to estimate the effective masses m_{CB}^* and m_{CF}^*. Substitute $n = 0.87 \times 10^{11} cm^{-2}$, $\alpha^H = 0.03K^{-1}$ and $\alpha^L = 0.19K^{-1}$ into eq (7) and its counterpart for CFs in the HF transition, one can get,

$$m_{CB}^* \approx 9m_b, \qquad m_{CF}^* \approx 6m_b,$$

which give reasonable values of several band mass for $m_b = 0.1m_e$. This fact gives support to our usage of the CB(CF) picture in the LF(HF) transition.

Then let us turn to the discussion of the critical longitudinal resistivity ρ_c^H and ρ_c^L. At the critical point of the HF(LF) transition, the effective magnetic field B^* is averaged to zero. To get started, we adopt the simplest picture of free CFs (CBs) moving in a random potential. This picture is not as easy as it seems to be, because

the disorder is relatively strong. (From the measurement, $\rho_c \approx 2.2h/e^2$, so $k_F l$ is of the order of 1 or less, which has reached the IR limit). Therefore, for the case of CFs, we can expect the Drude formula to hold at most marginally, which gives:

$$\begin{aligned}\sigma_c^{CF} &= ev_F \frac{dn}{dE}(el_{CF}) \qquad (8)\\ &= ev_F \frac{n}{E_F}(el_{CF})\\ &= ev_F \frac{k_F^2}{4\pi E_F}(el_{CF})\\ &\propto \frac{e^2}{h}(k_F l_{CF}).\end{aligned}$$

One can see that ρ_c^H, or its inversion σ_c^{CF} is uniquely determined by a single dimension-less parameter $k_F l_{CF}$, which measures the extent of disorder. We then make a reasonable extension of the above conclusion to the case of CBs, with k_F substituted by the typical wave vector k_{CB} specific to the CBs and l_{CF} substituted by its CB counterpart l_{CB}, We note that k_{CB} is much smaller than k_F at a temperature $T << E_F$ (we avoid applying the Drude formula directly to the CBs, because its wavelength is much larger than l_{CB}, and the classical picture is no longer valid). If we suppose that the mean free path is almost the same for CFs and CBs, then this dimensionless parameter for CBs will be much smaller than that for CFs. Therefore the localization effect of disorder is more severe on CBs than on CFs. So we can expect the disorder potential induced effective mass for CBs to be larger than its counterpart for CFs. This is consistent with above calculation of $m_{CB}/m_{CF} \approx 1.5$.

The above "single parameter" argument can also be applied to the qualitative analysis of the temperature-dependences of the critical resistivities ρ_c^H and ρ_c^L. Let us first suppose a similar T-dependence in the mean free paths for CFs and CBs (that is, they decrease as T increases). Then for the CF case, only CFs near the Fermi surface have contributions to the transport properties, with momentum k_F almost independent of T. So the parameter $k_F l_{CF}$ will decrease as T increases, which implies that ρ_c^H will increase as T goes up. In contrast, for the case of CBs, an increase in T will excite the CBs from low momentum states to higher momentum ones, which results in a considerable increase in k_{CB} that can counteract the decrease in l_{CB} [13], so the overall tendency for the parameter $k_{CB} l_{CB}$ will be an increment. Therefore ρ_c^L will increase as T goes up. In this way, one can see that the opposite T-dependences in ρ_c^H and ρ_c^L come from the presence of a Fermi surface in the CFs and the absence of one in the CBs, which has its origin from the opposite statistical natures of fermions and bosons. Based on this speculation, we suggest doing the same measurement on a sample with its density 10 times smaller and in the same range of temperature(0K to 10K). In this case, E_F will be of the same order of T, and the Fermi surface effect will be weakened considerably. Therefore the different T-dependences between ρ_c^H and ρ_c^L should disappear.

Then we comment briefly on the relation between ρ_c^H and ρ_c^L at $T = 0$. The possible universal relation $\rho_c^H(T=0) = \rho_c^L(T=0)$, as suggested by Hilke at el. can not be understood easily in the present picture, because it is difficult to give a reliable analytical equation for a strongly disordered, non-interacting bosonic system. Numerical methods for model calculations are suitable in this respect, which will be the focus of our future work.

We then turn to a tentative discussion of the origin of the zero-temperature dissipative conductivity reflected from the non-zero β^H and β^L. According to our phenomenological picture, near the HF (LF) transition, CFs (CBs) move under an effective magnetic field B^*. In the single particle approximation, the CFs or CBs reside on the nearly localized quantum Hall states whose spatial distributions are proportional to $\exp(-(x-X)^2/2l_{B^*}^2)$ with the center X distributes almost uniformly across the plane, where l_{B^*} is the magnetic length corresponding to B^*. At the zero temperature limit, we expect quantum tunnelings between the quantum Hall states to dominate the transport properties. Therefore we suggest that the average tunneling probability $p(T=0)$ which is proportional to $\sigma_{xx}(T=0)$, is determined by

$$p(T=0) \propto \exp(-d^2/2l_{B^*}^2)$$

where d is the average distance between adjacent particles, or $d \approx 1/\sqrt{n}$. Then by using the following relations:

$$l_{B^*} = \sqrt{\frac{\hbar}{eB^*}}, \quad \nu^* = nh/eB^*, \quad \nu^* = \frac{\nu\nu_c}{|\Delta\nu|}$$

we can easily arrive at

$$\sigma_{xx}(T=0) \propto \exp(-\pi\frac{|\Delta\nu|}{\nu_c^2})$$

Therefore we can estimate

$$\beta \approx \frac{\nu_c^2}{\pi}$$

One can compare the above result with the experimentally determined β^H and β^L by substituting $\nu_c = 1/2$ and 1 for the HF and LF transitions respectively. The theoretical values are

$$\beta^H \approx 0.08, \quad \beta^L \approx 0.3,$$

which give a surprisingly good fit with the experimental data (see Fig.2). We comment that in Fig.2b the residue value of α_0 should be much closer to 0.3 considering the flattening tendency of the dots when approaching $T = 0$. As we believe, the above consistency should be a very

strong support to our seemingly naive understandings based on CF and CB respectively.

Before closing, let us give the following comments in order. First, let us comment on the metallic T-dependence for ρ_c^H, which is described here in the framework of free CFs in a random potential without a magnetic field. According to the conventional belief [14], a two dimensional non-interacting system will be localized to an insulator upon the introduction of an infinitesimal extent of disorder, so the metallic phase is absent. However, the situation here is quite different. Because of the gauge fluctuations in the CF system which break the time-reversal symmetry, the weak localization effect [15] that leads to the localization is suppressed. Therefore it is still possible for the CF system to demonstrate metallic behaviors at the limit $k_F l_{CF} \approx 1$. Then, let us discuss the role of the Coulomb interaction in the CF system. In a low-density disordered two dimensional system, Coulomb gap has important consequences in transport properties(i.e. a hopping conductivity $\approx \exp(-\sqrt{\frac{T_0}{T}})$) [16]. But the effect of Coulomb interaction is different for CFs. Since the particles are confined to the lowest LL, the CF effective mass has its source from interaction and disorder. In Read's recipe [17], the residue interactions between the CFs (the almost neutral flux-hole-electron triplet) are reduced considerably. So the effect of Coulomb interaction is mainly absorbed into the effective mass of the CFs. To be complete, we do not exclude the interaction induced $\ln(T)$ correction to the conductivity [18], which is supposed to be important at quite low temperature and irrelevant to the experiment here(the T-dependence of ρ_c^H manifests for T larger than 3K). As for the CB system, we believe that the above comments are probably also applicable. Finally, we would like to suggest that both CFs and CBs are good quasiparticles in quantum Hall systems, and they are not simply equivalent to each other, nor do they exclude each other. They assume domination in different regimes of the phase diagram, with the excitation energy scales close to the minimum. Because of the opposite statistical natures of CFs and CBs, we can expect many diverse properties to be observed, which will be the task of the future experimentalists.

In conclusion, we have extended a newly advanced CF picture of the high-field insulator-Hall liquid transition to its CB counterpart, which is then applied to the analysis of the low-field insulator-Hall liquid transition. We thus present a comparative study of these two transitions. In this way, the similar reflection symmetries in filling factors in both transitions are understood consistently as due to the symmetry of the gapful excitations which dominate σ_{xx} across the transitions, and the abrupt change in σ_{xy} at the transitions. The substantially different characteristic energy scales involved in these two transitions can be attributed to the differences in critical filling factors ν_c and the effective masses. The opposite temperature-dependences of the critical longitudinal resistivities are also well-understood, which can be traced to the opposite statistical natures of the composite fermion and the composite boson. We also give a tentative discussion of the zero-temperature dissipative conductivity, and arrive at a good fit with the experiment.

We acknowlege helpful discussions with Prof. Y.S.Wu. We are also thankful to Dr. Shahar for his kindly permission to our use of the figures. This work is partly supported by National Natural Science Foundation of China.

---

[1] S. Kivelson, D. H. Lee and S. C. Zhang, Phys. Rev. B **46**, 2223 (1992).

[2] For a review, see S. C. Zhang, Int. J. Mod. Phys. B **6**, 25 (1992).

[3] B. I. Halperin, P. A. Lee and N. Read, Phys. Rev. B **47**, 7312 (1993).

[4] J. K. Jain, Phys. Rev. Lett. **63**, 199 (1989).

[5] L. W. Engel, H. P. Wei, D. C. Tsui, and M. Shayegan, Surf. Sci. **229**, 13 (1990); L. W. Wong, H. W. Jiang, N. Trivedi and E. Palm, Phys. Rev. B **51**, 18033 (1995).

[6] B. Huckestien, Rev. Mod. Phys. **67**, 357 (1995).

[7] S. L. Sondhi, S. M. Girvin, J. P. Carini and D. Shahar, Rev. Mod .Phys. **69**, 315 (1997).

[8] H. W. Jiang, C. E. Johnson, K. L. Wang and S. T. Hannahs, Phys. Rev. Lett. **71**, 1439 (1993); T. Wang, K. P. Clark, G. F. Spencer, A. M. Mack, and W. P. Kirk, Phys. Rev. Lett. **72**, 709 (1994); B. W. Alphenaar and D. A. Williams, Phys. Rev. B **50**, 5795 (1994); D. Shahar, D. C. Tsui, M. Shayegan, R. N. Bhatt and J. E. Cunningham, Phys. Rev. Lett. **74**, 4511 (1995).

[9] M. Hilke, D. Shahar, S. H. Song, D. C. Tsui, Y. H. Xie and Don Monroe, cond-mat/9708239.

[10] D. Shahar, D. C. Tsui, M. Shayegan, E. Shimshoni and S. L. Sondhi, Science **274**, 589 (1996); D. Shahar, M. Hilke, C. C. Li and D. C. Tsui (cond-mat/9706045).

[11] D. E. Khmelnitskii, Phys. Lett. A **106**, 182 (1984); R. B. Laughlin, Phys. Rev. Lett. **52**, 2304 (1984); F. D. M. Haldane and Kun Yang, Phys. Rev. Lett. **78**, 298 (1997).

[12] Wenjun Zheng, Yue Yu and Zhaobin Su, cond-mat/9709225.

[13] To see this quantitatively, we present the following estimation: Suppose the CB's average kinetic energy is of the order $k_B T$, then we have $k_{CB} \sim \sqrt{T}$. It is reasonable to assume l_{CB} to be near its non-zero value at $T = 0$, if the temperature is low. So the product $k_{CB} l_{CB}$ is approximately proportional to $\sqrt{T}$.

[14] E. Abrahams, P. W. Anderson, D. C. Licciardello, and T. V. Ramakrishnan, Phys. Rev. Lett. **42**, 673 (1979).

[15] For a review, see P. A. Lee and T. V. Ramakrishnan, Rev. Mod. Phys. **57**, 287 (1985).

[16] A. L. Efros and B. I. Shlovskii, J. Phys. C. **8**, L49 (1975).

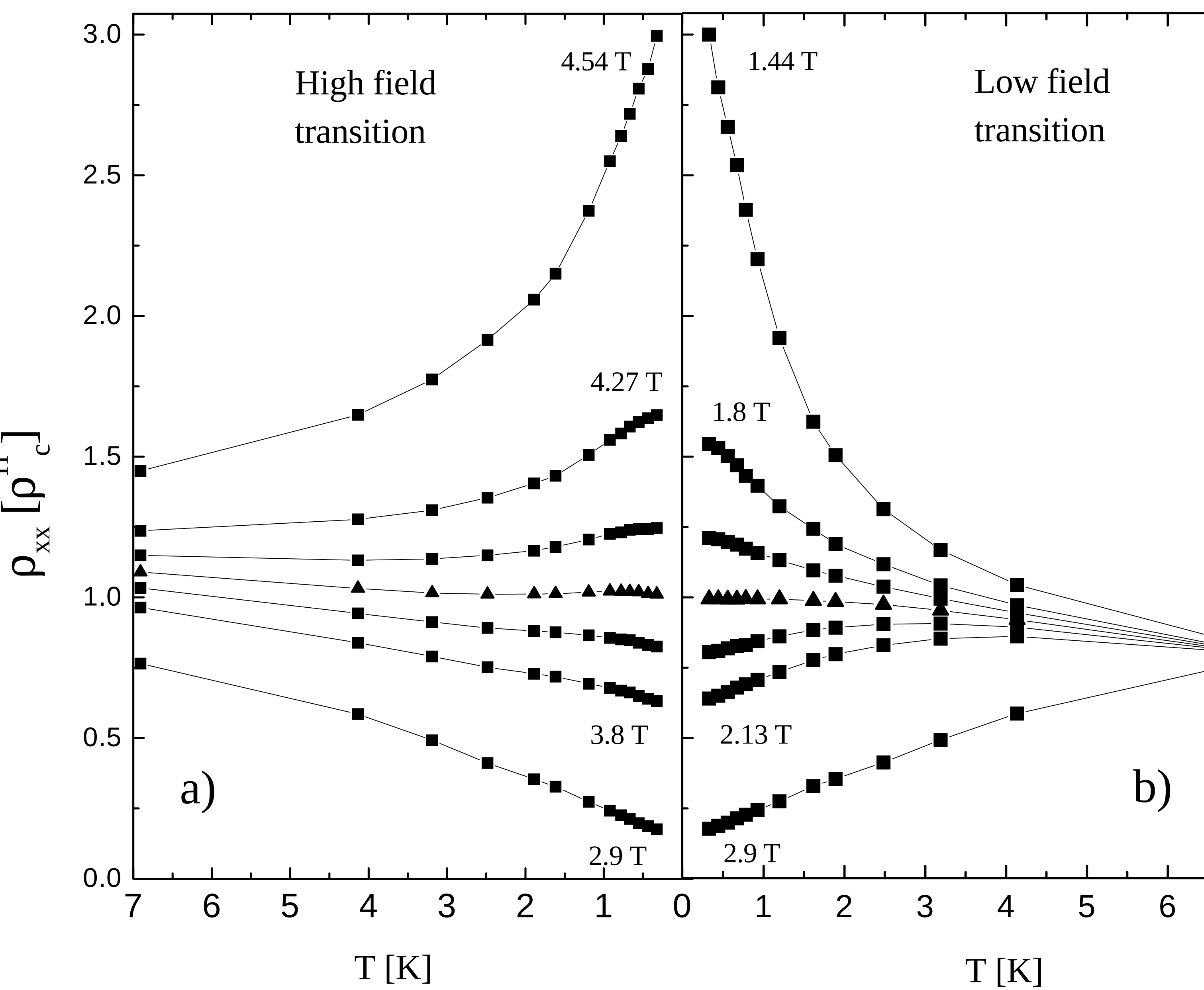
[17] N. Read, Semicon. Sci. Tech. **9**, 1859 (1994).

[18] D. V. Khveshchesko, Phys. Rev. Lett. **77**, 362 (1996); L. P. Rokhinson, B. Su and V. J. Goldman, Phys. Rev. B

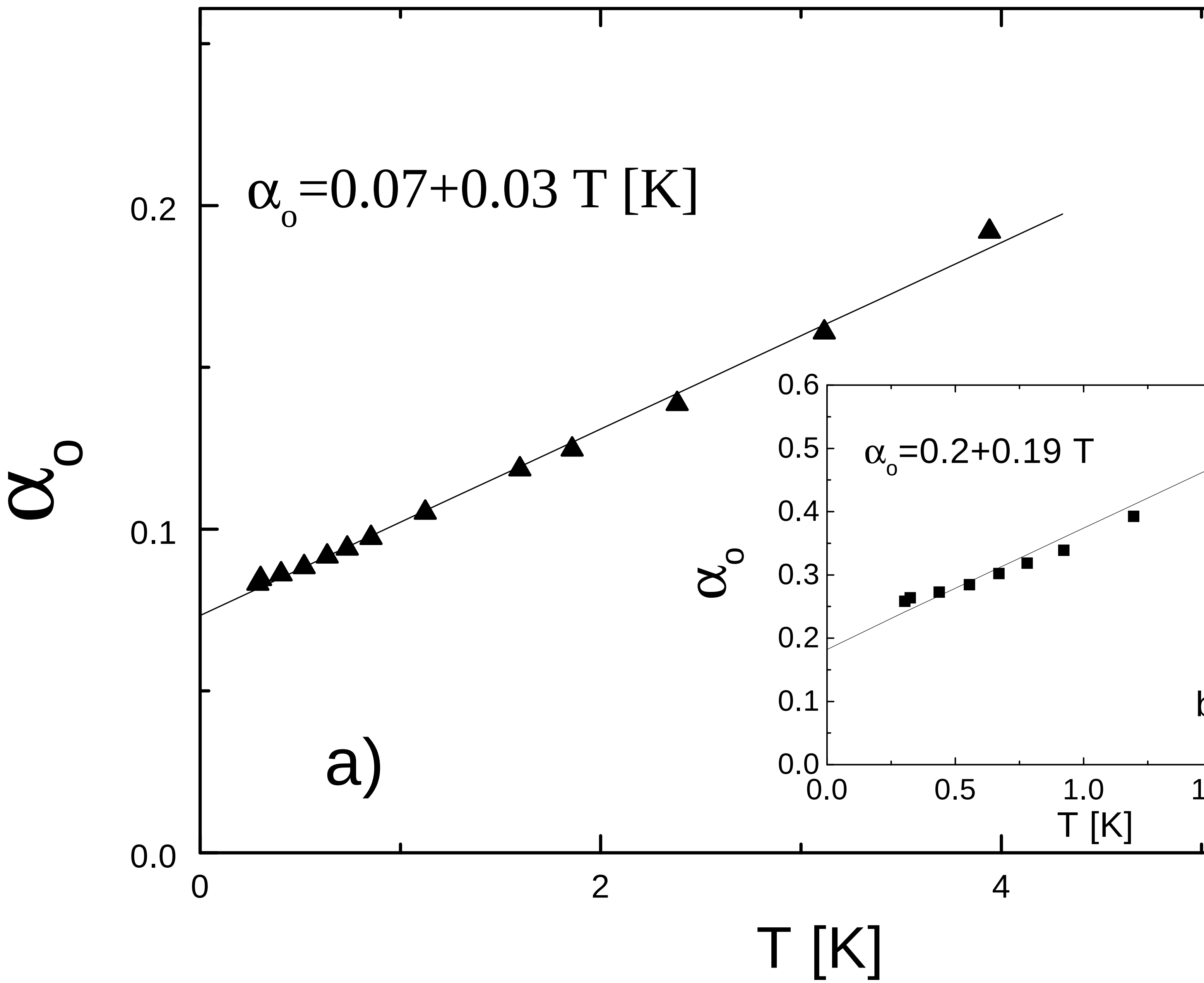
FIG. 1. Temperature dependence of the resistivities around the low and high field transitions. In fig. 2 a) the magnetic fields corresponding to the central resistivity curves are 3.94, 4.04 and 4.14 T and in fig. 2 b) they are 2.05, 1.975 and 1.9 T (reprinted from ref[9]).

FIG. 2. $\alpha_0(T)$ on a linear graph as a function of temperature T for the high-field transition. The inset shows the low-field transition up to 1.7 K. The straight lines are linear fits to the data (reprinted from ref[9]) .

52, R11588 (1995).



High field transition
Low field transition
4.54 T
4.27 T
3.8 T
2.9 T
1.44 T
1.8 T
2.13 T
2.9 T
a)
b)
ρ_{xx} [ρ^H_c]
T [K]
T [K]
3.0
2.5
2.0
1.5
1.0
0.5
0.0
7
6
5
4
3
2
1
0
1
2
3
4
5
6



α_o=0.07+0.03 T [K]
a)
α_o
T [K]
0.0
0.1
0.2
0
2
4
α_o=0.2+0.19 T
b
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.0
0.5
1.0
T [K]